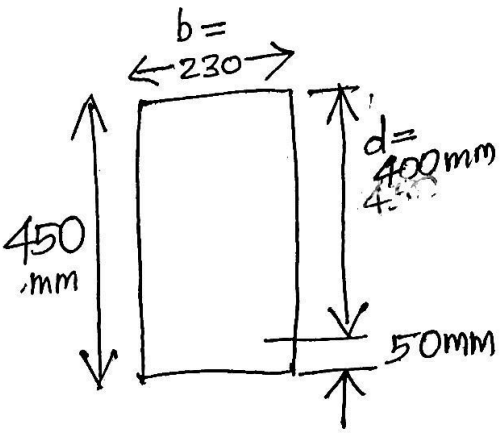
Q: Design an RCC beam of size 230×450mm to carry a service load of 20 kN/m, which is simply supported over effective span of 5.50 m. Use M20 concrete & Fe500 steel



Ans:

Simply supported beam

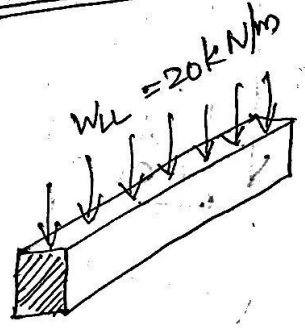
$l = 5.50\text{m}$

$D = 450\text{ mm}$

Assume effective cover = 50mm

Effective depth $d = D - 50$

$= 450 - 50 = \underline{400\text{ mm}}$



Total load on the beam:

$W_{LL} = 20\text{ kN/m}$

$W_{DL} = 25\text{ kN/m}^3 \times 0.23 \times 0.45$

$= 2.59\text{ kN/m}$

Total load $W = \underline{22.59\text{ kN/m}}$

Unit weight of RCC section

$= 25\text{ kN/m}^3$

(IS 875-part 1)

W_{DL} = Unit wt × c/s area

$= ____\text{ kN/m}$

Factored load $W_u = 1.5\,W$

$= 1.5 \times 22.59$

$= 33.89\text{ kN/m}$

Factored moment (M_u) $= \frac{W_u l^2}{8} = \frac{33.89 \times 5.5^2}{8}$

$= 128.15\text{ kNm}$

$M_u = 128.15$ kNm

Limiting moment of resistance of beam:

$$M_{ulim} = 0.36 f_{ck} b x_{umax} (d - 0.42 x_{umax})$$

$$= 0.36 \times 20 \times 230 \times 0.46 \times 400 \left(400 - 0.42 \times 0.46 \times 400\right)$$

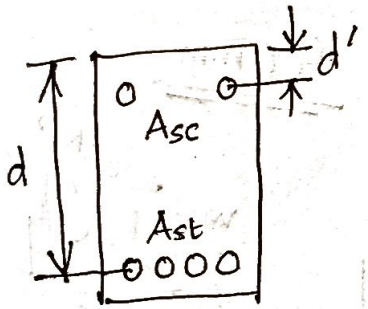
$$= 98.33 \times 10^6 \text{ Nmm}$$

$$= 98.33 \text{ kNm}$$

$f_{ck} = 20 \text{ N/mm}^2$
$b = 230$ mm
$d = 400$ mm
$x_{umax} = 0.46d$
$= 0.46 \times 400$
(Fe 500 grade)

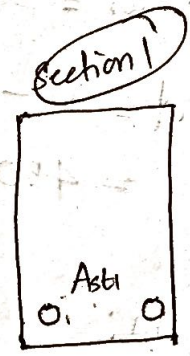
So $M_u > M_{ulim}$

$\Rightarrow$ Doubly Reinforced beam should be designed



(resists M_u)

$=$



(resists M_{ulim})

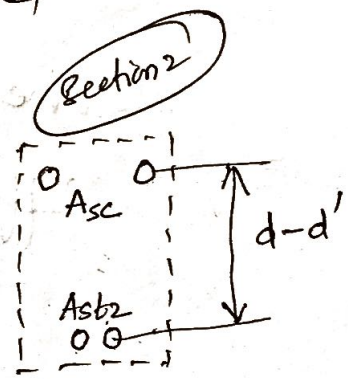
[BALANCED SECTION]
(Singly reinf beam)

$C = 0.36 f_{ck} b x_{umax}$

$T = 0.87 f_y A_{st1}$

$z = d - 0.42 x_{umax}$

$+$



(resists $M_u - M_{ulim}$)

$C = f_{sc} A_{sc} - 0.45 f_{ck} A_{sc}$

$T = 0.87 f_y A_{st2}$

$z = d - d'$

$A_{st} = A_{st1} + A_{st2}$

Section 1

$$M_{ulim} = T \cdot z$$
$$= 0.87 f_y A_{st1} (d - 0.42 x_{umax})$$

$$98.33 \times 10^6 = 0.87 \times 500 \times A_{st1} (400 - 0.42 \times 0.46 \times 400)$$

$$\Rightarrow A_{st1} = \underline{700.44 \text{ mm}^2}$$

Section 2

$$M_u - M_{ulim} = T \cdot z$$
$$= 0.87 f_y A_{st2} (d - d')$$

$$(128.15 - 98.33) \times 10^6 = 0.87 \times 500 A_{st2} (400 - 50)$$

$$\Rightarrow A_{st2} = \underline{195.86 \text{ mm}^2}$$

$\therefore$ Tension steel area of the beam

$$A_{st} = A_{st1} + A_{st2} = 700.44 + 195.86$$
$$= \underline{897 \text{ mm}^2}$$

$$M_u - M_{ulim} = C \cdot z$$
$$= [f_{sc} A_{sc} - 0.45 f_{ck} A_{sc}] (d - d')$$

$\Rightarrow$ $(128.15 - 98.33) \times 10^6$
$= (403.50 A_{sc} - 0.45 \times 20 A_{sc})$
$(400 - 50)$

$\Rightarrow A_{sc} = 215.97 \text{ mm}^2$

To find f_{sc}, follow Table F (SP16)

$d'/d = \frac{50}{400} = 0.125$

$d'/d = 0.1 \rightarrow f_{sc} = 412 \text{ N/mm}^2$
$d'/d = 0.15 \rightarrow f_{sc} = 395 \text{ N/mm}^2$

f_{sc} for $(d'/d = 0.125)$
$= \frac{412 + 395}{2}$
$= 403.50 \text{ N/mm}^2$

Area of compression steel = $215.97\ mm^2$

$\approx 216\ mm^2$

Limits of Ast:

$A_{st,min} = 0.85\frac{bd}{f_y} = \frac{0.85 \times 230 \times 400}{500} = 156\ mm^2$

$A_{st,max} = 0.04\ bD = 0.04 \times 230 \times 450 = 4140\ mm^2$

A_{st} obtained $= 897\ mm^2$ ✓

OK

Limits of Asc

$A_{sc,min} = 0$

$A_{sc,max}$ = same as $A_{st,max} = 0.04bD = 4140\ mm^2$

A_{sc} obtained $= 216\ mm^2$ ✓

OK

$A_{st} = 897\ mm^2$

$A_{sc} = 216\ mm^2$

Using 16 mm ϕ for tension reinforcents

$$\text{No of bars} = \frac{897}{\pi/4 \times 16^2} \approx 5 \text{ bars}$$

Provide 5 bars of 16 mm ϕ on tension side

Using 16 mm ϕ bars on compression side

$$\text{No of bars} = \frac{216}{\pi/4 \times 16^2} \approx 2 \text{ bars}$$

Provide 2 bars of 16 mm ϕ on compression side.

Depth of neutral axis :

$$x_u = \frac{0.87 f_y A_{st} - f_{sc} A_{sc} + 0.45 f_{ck} A_{sc}}{0.36 f_{ck} b}$$

$$= \frac{\left(0.87 \times 500 \times \left(5\ \pi/4 \times 16^2\right)\right) - \left(403.50 \times 2\pi/4 \times 16^2\right) + \left(0.45 \times 20 \times 2\pi/4 \times 16^2\right)}{0.36 \times 20 \times 230}$$

$$= 168.20 \text{ mm}$$

$$x_{umax} = 0.46\, d = 0.46 \times 400 = 184 \text{ mm}$$

Since $x_u < x_{umax} \Rightarrow$ Under-reinforced section

$\therefore$ Safe